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Hadwiger 予想
Hadwiger conjecture

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Abstract: Let $k \in \mathbb{N}$. We prove that a graph G with no $(k+1)$-clique minor is k-colorable. The problem is well-known (see e.g. [5]). It was posed by H. Hadwiger in 1943. The case that $k = 4$ is equivalent to four-color theorem (see [6]). Our problem is deeply fundamental and almost no known concepts may succeed to make a solution. We thus use a nondeterministic algorithm, from which the proof is critically simplified.

Keywords: Hadwiger conjecture

1 Introduction

Let $k \in \mathbb{N}$. We prove the following theorem:

Theorem 1 (Hadwiger conjecture)**.** *A graph G with no $(k+1)$-clique minor is k-colorable.*

The problem is well-known (see e.g. [5]). It was posed by H. Hadwiger in 1943. The case that $k \leq 5$ is proved (see [3], [1], [2], [4]) and the case that $k = 4$ is equivalent to the celebrated four-color theorem, which states that every planar map is four colorable (see [6]). Theorem 1 is deeply basic and almost no known concepts may succeed to give a solution. We thus use a nondeterministic algorithm, which critically simplifies the proof.

2 Proof of Theorem 1

Let $\chi(G)$ be the chromatic number of G. Let $h(G)$ be the Hadwiger number of G.

Proof of Theorem 1. It is proved by induction on the number of the vertices involved (the ones adjacent to P involved) that if there exists a vertex P such that P and $\chi-1$ vertices of G adjacent to P are of the number of the minimal colors equal to χ then they (i.e. the $\chi-1$ vertices of G adjacent to P) are connected one another by paths in S, where S is a graph obtained from G by deleting P, and the paths are not by way of the other vertices among themselves and without common vertices in each pair.

From above it is easy to prove that we may obtain a $\chi(G)$-clique by taking minors of G. Thus $\chi(G) \leq h(G)$. The assertion follows. □

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